Solutes in Fixed Beds: Pure Thermal Wave Formation and Its Practical Implications," *Chem. Eng. Sci.*, **25**, p. 1653 (1970).
Pan, C. Y., and D. Basmadjian, "An Analysis of Adiabatic Sorption of Single Solutes in Fixed Beds: Equilibrium Theory," *Chem. Eng. Sci.*, **26**, p. 45 (1971).
Rhee, H. K., and N. R. Amundson, "An Analysis of an Adiabatic Adsorption Column: Part I. Theoretical Development," *Chem. Eng. J.*, **1**, p. 241 (1970).
Rhee, H. K., E. D. Heerdt, and N. R. Amundson, "An Analysis of an Adiabatic Adsorption Column: Part II. Adiabatic Adsorption of a Single Solute," *Chem. Eng. J.*, **1**, p. 279 (1970).
Sutherland, J. W., P. J. Banks, and H. J. Griffiths, "Equilibrium Heat and Moisture Transfer in Air Flow Through Grain," *J. Agric. Eng. Res.*, **16**, p. 368 (1971).

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Hold-Up Studies in the Hydraulic Conveying of Solids in Horizontal Pipelines

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INTRODUCTION

Hydraulic transportation of solids is an economical, efficient and reliable method of transport particularly in high-volume long-distance hawls. Many projects around the globe are either operating or are at varying stages of development. Recent completion of 60 km Kudremukh iron ore pipeline transportation project in India marks the beginning of successful large-scale application of hydraulic transport in this country.

In view of the commercial utility of such transportation systems a number of correlations have been developed for predicting pressure loss and critical velocity. However, almost all the available correlations assume the slip velocity between solid and fluid to be zero, so that *in-situ* porosity equals that at the delivery end. This can be grossly erroneous as is illustrated by present experimental results and those of Kao et al. (1980) in Figure 1.

Gandhi (1976) pointed out that the differences in the predictions of the various correlations could be due to the difference between *in-situ* and delivered porosities. He proposed a method to predict hold-up inside a horizontal transport pipeline. In this method, solid velocity distribution was assumed independent of terminal settling velocity of particles. Consequently, even for zero slip velocity the reuslts gave *in-situ* porosity to be different from delivered porosity. This contradicts the basic material balance equations (Eq. 5).

Spedding and Nguyen (1978) have proposed a general theory for prediction of hold-up and arrived at an equation having two parameters which are to be experimentally obtained. Hence this equation cannot be used as such to predict *in-situ* porosity from the known operating conditions. Furthermore, even their equation predicted *in-situ* and delivered porosities to be different for homogeneous transport. Since homogeneous transport can strictly occur only if the slip velocity is zero, the validity of their equation is equally questionable as it would also contradict the material balance equation, Eq. 5.

Televentos et al. (1979) clearly pointed out that one relationship, that of *in-situ* porosity, was missing from the set of equations that could make their model complete. They overcame this difficulty by assuming a constant solid concentration in the lower layer of their two-layer model.

The preceding discussion clearly shows that a method of predicting *in-situ* porosity from known conditions would be highly useful. An attempt is made in this paper to develop a semitheoretical mathematical model for this purpose having a single parameter. From experiments covering a limited range, this parameter has been correlated with system variables.

DEVELOPMENT OF EQUATION

The flow of slurry is assumed to be one-dimensional. The flow parameters are assumed to be constant throughout the length of the pipeline. The quantity of interest is only the average porosity at any cross section. Then the flow rates of fluid and solid through the pipeline are related to voidage or porosity inside the pipe by

$$Q_f = A\epsilon v_f \quad (1)$$

$$Q_s = A(1-\epsilon)v_s \quad (2)$$

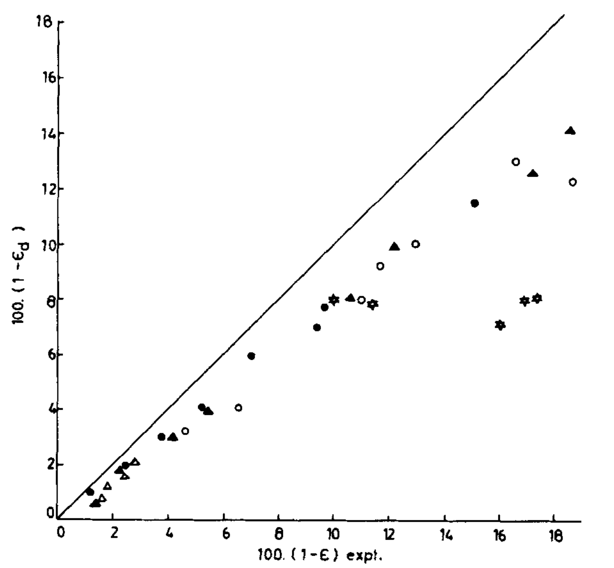

Figure 1. Comparison of *in-situ* and delivered solid concentration in hydraulic transportation of sand. For legend, see Table 1.

The porosity at the delivery end is given by

$$\epsilon_d = \frac{Q_f}{(Q_f + Q_s)} = \frac{\epsilon}{\epsilon + (v_s/v_f)(1 - \epsilon)} \tag{3}$$

The mixture velocity inside the pipeline is simply

$$v_m = Q/A = (Q_s + Q_f)/A \tag{4}$$

In the first *three* equations, Q_s and Q_f are process requirements and are hence known and the unknowns number *four* (v_s, v_f, ϵ, and ϵ_d). In principle, *in-situ* porosity should be predictable but there does not exist any theoretical equation which can predict it directly. The purpose of this paper is to give this fourth relationship which is required not only to solve for the four unknowns (in Eqs. 1 to 3) but which is also required for predicting pressure drop. Rearrangement of Eq. 3 leads to $\epsilon\epsilon_d + (v_s/v_f)\epsilon_d - (v_s/v_f)\epsilon\epsilon_d = \epsilon$ which can be expressed as

$$\left.\begin{aligned} \epsilon/\epsilon_d &= a/(1 - \epsilon_d + a\epsilon_d) \\ (1 - \epsilon)/(1 - \epsilon_d) &= 1/(1 - \epsilon_d + a\epsilon_d) \end{aligned}\right] \tag{5}$$

where

$$a = v_s/v_f \tag{6}$$

Since ϵ has to equal ϵ_d if there is no slip between particle and fluid, i.e., if a is unity, the possible correlation could be

$$\epsilon/\epsilon_d = a^{1-n} \tag{7}$$

Equations 5 and 7 give

$$n = ln[1 - (1 - a)\epsilon_d]/ln\ a \tag{8}$$

Since $(1 - a)$ and $(1 - a)\epsilon_d$ are always bounded by zero and unity, Eq. 8 can be expanded in Taylor series and subsequently rearranged to give

$$n = \epsilon_d \frac{[1 + (1 - a)\epsilon_d/2 + (1 - a)^2\epsilon_d^2/3 \ldots\ldots]}{[1 + (1 - a)/2 + (1 - a)^2/3 \ldots.]} \tag{9}$$

It is noteworthy that ϵ_d is outside the brackets in Eq. 9. This makes the variation in $b(= n/\epsilon_d)$ to be extremely small compared to the variation in the n values. Hence the endeavor should be to correlate b rather than n with the system variables.

From Eqs. 5 to 7, porosity inside the pipe ϵ is given by

$$\left.\begin{aligned} \epsilon^{b\epsilon_d}(1 - \epsilon)^{1-b\epsilon_d} &= \epsilon_d^{b\epsilon_d}(1 - \epsilon_d)^{1-b\epsilon_d} \\ \text{or } f(\epsilon) &= f(\epsilon_d) \end{aligned}\right] \tag{10}$$

which can be easily solved by any simple numerical technique. It may be mentioned here that one of the roots of Eq. 10 is ϵ_d itself, the other root being ϵ. However, for b very near to unity, it can be proved using L'Hospital rule that

$$\epsilon = \epsilon_d(2b - 1) \tag{11}$$

$$(1 - \epsilon) = (1 - \epsilon_d) + 2\epsilon_d(1 - b) \tag{12}$$

For all the data corresponding to present experiments, the above approximation deviates from the exact value obtained from the numerical solution of Eq. 10 by a maximum of 3% only.

EXPERIMENTAL

The apparatus used for hydraulic conveying studies has been reported by Wani (1982). The total length of the pipeline was 8 m. Delivered concentrations were calculated from the slurry volume and the dried sand volume collected for a known amount of time. The solids feed valve was then suddenly closed while allowing water to flow through to drain out the solids remaining in the pipeline. From the quantity of solids thus collected and the volume of the pipeline *in-situ* porosity was calculated. This assumes that the porosity throughout the length of the pipe is a constant. The operating conditions studied are summarized in Table 1.

ANALYSIS OF EXPERIMENTAL DATA

The b value has to be calculated from experimentally measured ϵ and ϵ_d values, and this has to be correlated to design and operating variables. For flow tending towards homogeneous conditions, b value should tend towards unity. Since homogeneous transport can occur if d_p or u_t tends towards zero, or if v_f tends to infinity, $(1 - b)$ would tend towards zero under these conditions. Further, since b always stays near to unity the variation in b about its mean value is negligible whereas the variation in $(1 - b)$ about its mean value is considerable. Due to these reasons, $(1 - b)$ is chosen as the dependent variable. Dimensional analysis leads to

TABLE 1. HYDRAULIC TRANSPORTATION OF SAND

References	d_p μm	Symbol	Other Details
Present Work	263	△	$D = 2.5$ cm $\rho_p = 2.65$ g/cc
	355	▲	$\rho = 1.0$ g/cc $Q_s/A = 2.0$ to 35 cm/s
	450	●	$v_m = 1.5$ to 3.0 m/s
	507	○	
Kao et al. (1980)	600	☆	$D = 5.0$ cm $Q_s/A = 16.0$ to 21.0 cm/s

$$(1 - b) = f(g,D,d_p,\rho,(\rho_p - \rho),\mu,Q_s,Q) \tag{13}$$

The dimensionless groups obtained are $Ga(= d_p^3 g\rho(\rho_p - \rho)/\mu^2)$, (ρ_p/ρ), (Q_s/Q), (d_p/D), and $(Q/D^2)/\sqrt{gD}$. Since the Galileo number is uniquely related to the Reynolds number ($Re = d_p u_t \rho/\mu$), Re is used instead of Ga. The influence of (d_p/D) is assumed to be negligible. The density ratio and the flow ratio are assumed to influence in a combined manner (for simplicity) as $N = Q_s\rho_p/(Q\rho)$. The last number is nothing but the Froude number multiplied by a constant factor ($= 4/\pi$). Therefore, the equation trend is

$$(1 - b) = K_1 Re^{K_2} Fr^{K_3} N^{K_4} \tag{14}$$

where

$$Re = d_p u_t \rho/\mu;\ Fr = v_m/\sqrt{gD};\ N = Q_s\rho_p/(A\rho v_m) \tag{15}$$

The coefficients in Eq. 14 were obtained by logrithmic linear regression. The final correlation obtained is

$$(1 - b) = 18\left(\frac{v_m}{\sqrt{gD}}\right)^{-3.5}\left(\frac{Q_s}{Av_m}\right)^{1.5}\left(\frac{\rho_p}{\rho}\right)^{1.5} \tag{16}$$

Equations 3, 4 and 16 yield

$$(1 - b) = 18(1 - \epsilon_d)^{1.5}\left(\frac{v_m}{\sqrt{gD}}\right)^{-3.5}\left(\frac{\rho_p}{\rho}\right)^{1.5} \tag{17}$$

It is noteworthy that $(1 - b)$ is independent of Re. The details are not presented here as the prediction of $(1 - b)$ is not so important as that of $(1 - \epsilon_d)$. The prediction of $(1 - \epsilon_d)$ from Eqs. 12 and 17 are compared with experimental data in Figure 2. The average absolute deviation in Figure 2 is about 7% whereas the corresponding error in the assumption that *in-situ* and delivered solid concentrations are equal as shown in Figure 1 is 25%. The present correlation is valid for $(1 - \epsilon_d)$ up to 15% and $(1 - \epsilon)$ up to 20%).

The present experiments were carried out in a pipe of single diameter, 2.5 cm. Kao et al. (1980) have given *in-situ* solid concentrations measured in a pipe of 5 cm diameter. Their data points were not used to obtain the correlation, Eq. 17. Still, the predictions compare well with their data. The good comparison, therefore, confirms the pipe diameter dependence given by Eq. 17. Furthermore, since particle size in Kao et al.'s (1980) experiments was different from those in the present experiments, it gives support to the independence of $(1 - b)$ on Re obtained in Eq. 17.

CONCLUSIONS

It has been demonstrated that the delivered and *in-situ* concentration of solids in a horizontal hydraulic transportation pipeline

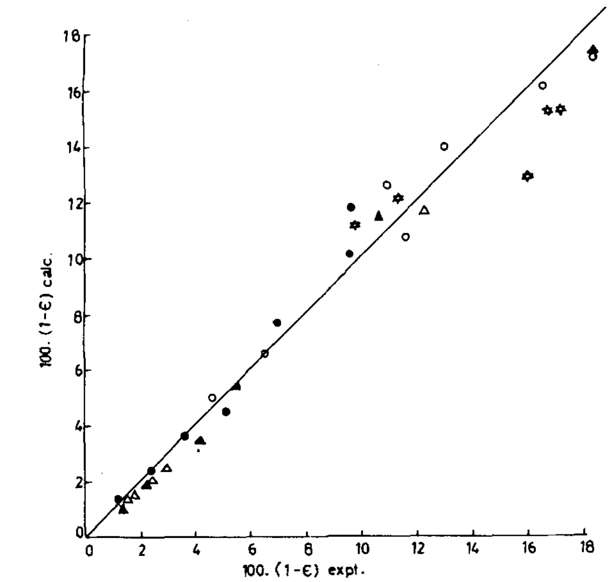

Figure 2. Comparison of experimental and predicted *in-situ* solid concentration. For legend, see Table 1.

can differ substantially which is due to the inherent slip existing between the solid and the fluid. The value of v_s/v_f varied between 0.6 and 1 in the present experiments. In any pressure drop correlation this fact has to be incorporated.

The *in-situ* solid concentration can be obtained from Eqs. 12 and 17. It must be emphasized, however, that the range of experimental conditions on which Eq. 17 is based is not very wide. However, till a comprehensive relation valid over a wide range of operating conditions is developed, Eqs. 12 and 17 can serve as useful first approximations in the design of hydraulic transportation systems.

NOTATION

a	= ratio of solid to fluid velocity, dimensionless
A	= area of cross section of the pipe, cm^2
b	= the ratio n/ϵ_d, dimensionless
d_p	= particle diameter, cm
D	= pipe diameter, cm
Fr	= $v_m/\sqrt{gD}$, dimensionless
g	= acceleration due to gravity, cm/s^2
Ga	= $d_p^3 g\rho(\rho_p - \rho)/\mu^2$, dimensionless
n	= exponent in Eq. 7, dimensionless
N	= defined in Eq. 15, dimensionless
Q	= volumetric slurry flow rate, equals $(Q_s + Q_f)$, cm^3/s
Q_f	= volumetric fluid flow rate, cm^3/s
Q_s	= volumetric solid flow rate, cm^3/s
Re	= $d_p u_t \rho/\mu$, dimensionless
u_t	= terminal settling velocity of particle, cm/s
v_f	= actual fluid velocity inside the pipe, cm/s
v_s	= actual solid velocity inside the pipe, cm/s
v_m	= mixture velocity, Q/A, cm/s

Greek Letters

ϵ	= *in-situ* fluid voidage or porosity, dimensionless
ϵ_d	= delivered fluid voidage or porosity, dimensionless
μ	= fluid viscosity, g/(cm·s)
ρ	= fluid density, g/cm^3
ρ_p	= particle density, g/cm^3
$(1 - \epsilon)$	= *in-situ* concentration of solids, dimensionless
$(1 - \epsilon_d)$	= delivered concentration of solids, dimensionless

LITERATURE CITED

Gandhi, R. L., "An Analysis of Hold-Up Phenomena in Slurry Pipelines," *Hydrotransport 4*, BHRA, A3-33 (1976).

Kao, D. T., A. Nosrati, and J. L. Schaefer, "Flow Behaviour of Solid-Liquid Mixtures in Pipes on Positive and Negative Grades," *Hydrotransort 7*, BHRA, F1-215 (1980).

Spedding, P. L., and V. T. Nguyen, "Hold-Up in Fluid-Solid Two-Phase Flow," *Chem. Eng. J.*, **15**, p. 131 (1978).

Televentos, Y., C. Shook, A. Carleton, and M. Streat, "Flow of Slurries of Coarse Particles at High Solids Concentrations," *Can. J. Chem. Eng.*, **57**, p. 255 (1979).

Wani, G. A., "Hydraulic Transportation of Solids Through Horizontal Pipes," Ph.D. Thesis, Indian Institute of Technology, Delhi (1982).

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Extension of Nichols Chart for Identification of Open-Loop Unstable Systems

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Recently Deshpande (1980) described a technique for dynamic identification of open-loop unstable systems. His method involves first obtaining experimental values for the closed-loop magnitude ratio, M, and phase angle, α, by pulse or step test implemented on a system stabilized with an arbitrarily tuned feedback controller. The corresponding open-loop magnitude ratio, A, and phase lag, θ (which exhibit unstable characteristics) are then obtained from Nichols chart via back calculation. The back calculation procedure is essentially the reverse of normal chart reading, i.e., fixing numerous closed-loop data points on the Nichols chart by manual